Approximate tight-binding sum rule for the superconductivity related change of c-axis kinetic energy in multilayer cuprate superconductors

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We present an extension of the c-axis tight-binding sum rule discussed by Chakravarty, Kee, and Abrahams [Phys. Rev. Lett. **82**, 2366 (1999)] that applies to multilayer high-T_c cuprate superconductors (HTCS) and use it to estimate—from available infrared data—the change below T_c of the c-axis kinetic energy, $\langle H_c \rangle$, in $YBa_2Cu_3O_{7-\delta}$ ($\delta = 0.45, 0.25, 0.07$), $Bi_2Sr_2CaCu_2O_8$, and $Bi_2Sr_2Ca_2Cu_3O_{10}$. In all these compounds $\langle H_c \rangle$ decreases below T_c and except for $Bi_2Sr_2CaCu_2O_8$ the change of $\langle H_c \rangle$ is of the same order of magnitude as the condensation energy. This observation supports the hypothesis that in multilayer HTCS superconductivity is considerably amplified by the interlayer tunnelling mechanism.

There is a growing evidence that at least in some HTCS the effective out-of-plane (c-axis) kinetic energy, $\langle H_c \rangle$, decreases upon entering the superconducting (SC) state [1,2]. In this manuscript we address the question whether this decrease represents a significant (and perhaps the dominant) contribution to the condensation energy or whether it is merely a small byproduct of an in-plane pairing mechanism. It is obvious that a quantitative estimate of the change of $\langle H_c \rangle$ between the normal and the SC state, $\Delta\langle H_c \rangle = \langle H_c \rangle(T \approx T_c) - \langle H_c \rangle(T << T_c)$, is required in order to answer this interesting question.

Two different approaches have been recently employed to obtain the value of $\Delta\langle H_c \rangle$: an approximate one [3] where $\Delta\langle H_c \rangle$ is identified with the Josephson coupling energy ("JCE", E_J) of the internal Josephson junctions and a rigorous one based on the so called c-axis tight-binding sum rule [4]. Within the JCE approach, the value of $\Delta\langle H_c \rangle$ per unit cell is given by the formula

$$\Delta\langle H_c \rangle = E_J = \frac{\hbar^2 \varepsilon_0 a^2}{4e^2 d} \omega_{pl}^2 \tag{1}$$

which contains only one nontrivial parameter, the plasma frequency of the internal Josephson plasmon ω_{pl}. The values of the in-plane lattice constant a and the distance between the neighboring copper-oxygen planes d are well known. The sum-rule (SR) approach instead relates $\Delta\langle H_c \rangle$ to the increase below T_c of the low-frequency optical spectral weight (SW)

$$\alpha(T, \omega) = \int_0^\omega \sigma_{1c}(T, \omega')\, \mathrm{d}\omega' \, , \tag{2}$$

$$\Delta\langle H_c \rangle = \frac{2\hbar^2 a^2}{\pi e^2 d}[\alpha(T << T_c, \Omega_c) - \alpha(T \approx T_c, \Omega_c)] = \frac{2\hbar^2 a^2}{\pi e^2 d}\Delta\alpha \, . \tag{3}$$

Here Ω_c is a cutoff frequency and σ_{1c} is the real part of the c-axis conductivity σ_c. What is the connection between the two approaches? The JCE approach yields only the contribution of the condensate (related to the spectral weight ρ_s of the δ peak at $\omega = 0$ in $\sigma_{1c}(\omega)$) but neglects the contribution due to the single particle tunnelling that is related to the change ΔN of the finite frequency part $N(T, \Omega_c)$ of $\alpha(T, \Omega_c)$. The SR approach, on the other hand, considers both contributions to $\Delta\langle H_c \rangle$. Only in the limit of vanishingly small changes at T_c of the regular part of σ_c the two approaches can be expected to yield the same result (except for a factor of 4 as pointed out in Ref. [4]). Especially the underdoped cuprate HTCS are not very far from this limit where the JCE approach can be expected to yield a reasonable—better than order of magnitude—estimate of ΔH_c.

In a previous paper [5] we and our coworkers have reported the values of E_J of two compounds that have two copper-oxygen planes per unit cell (bilayer compounds): $YBa_2Cu_3O_{7-\delta}$ (Y-123) and $Bi_2Sr_2CaCu_2O_8$ (Bi-2212). Note that in these compounds E_J is determined mainly by the frequency ω_{bl} of the intra-bilayer Josephson plasmon [6–11]

and by the distance between the closely spaced copper-oxygen planes. We have shown that there is a reasonably good agreement between the values of E_J and the values of the condensation energy U_0 obtained from the specific heat data. Assuming $\Delta\langle H_c\rangle \approx E_J$ we arrive at the conclusion that the condensation energy (and the high value of T_c) in the two compounds can be accounted for by the change at T_c of $\langle H_c\rangle$, i.e., by the interlayer tunneling theory [12]. Nevertheless, it can be objected that the actual value of $\Delta\langle H_c\rangle$ may be much smaller than the one of E_J and therefore should be determined by using the complementary and more rigorous SR approach. Unfortunately the approach in its simplest form as presented above does not apply to multilayer compounds [5]. Its derivation is based on the assumption that the distribution of the total internal electric field in the superconductor is homogeneous, a condition that is not met for multilayer systems. In the following we present an extension of the SR approach that allows one to obtain a reasonable estimate of $\Delta\langle H_c\rangle$ in multilayer cuprate HTCS. We provide a short derivation of the key formulas and discuss the applications to Y-123, Bi-2212, and $Bi_2Sr_2Ca_2Cu_3O_{10}$ (Bi-2223).

For the sake of simplicity we focus on the bilayer case and we use the notation introduced in Ref. [5], in particular in its appendix B. The effective c-axis kinetic energy (H_c) of a bilayer compound consists of the intra-bilayer term (H_{bl}) and the inter-bilayer one (H_{int}),

$$H_c = H_{bl} + H_{int} \text{ and } \Delta\langle H_c\rangle = \Delta\langle H_{bl}\rangle + \Delta\langle H_{int}\rangle. \tag{4}$$

The current densities in the intra-bilayer region and in the inter-bilayer region denoted as j_{bl} and j_{int}, respectively, are related to the two electric fields E_{bl} and E_{int} as follows (Eq. (B7) of Ref. [5]): $j_\alpha(\omega) = \sigma_{\alpha\beta}(\omega)E_\beta(\omega)$, α, $\beta \in \{bl, int\}$. It is reasonable to assume that the conductivity matrix $\sigma_{\alpha\beta}$ is diagonal and to introduce the abbreviations $\sigma_{bl} = \sigma_{bl\,bl}$ ("intra-bilayer conductivity") and $\sigma_{int} = \sigma_{int\,int}$ ("inter-bilayer conductivity") so that $j_{bl} = \sigma_{bl}E_{bl}$ and $j_{int} = \sigma_{int}E_{int}$. The total conductivity is then given as follows:

$$\sigma(\omega) = (d_{bl} + d_{int}) \Big/ \left[\frac{d_{bl}}{\sigma_{bl}(\omega)} + \frac{d_{int}}{\sigma_{int}(\omega)} \right], \tag{5}$$

which is the formula used at the phenomenological level introduced in Ref. [7]. We aim at finding a formula connecting $\Delta\langle H_c\rangle$ and $\Delta\alpha$ as defined in Eq. (3). Let us first express the two components $\Delta\langle H_{bl}\rangle$ and $\Delta\langle H_{int}\rangle$ of $\Delta\langle H_c\rangle$ in terms of the two conductivities. It follows from Eqs. of Appendix B of Ref. [5] that

$$\Delta\langle H_{bl}\rangle = \frac{2\hbar^2 a^2}{\pi e^2 d_{bl}}\Delta\alpha_{bl} \tag{6a}$$

and

$$\Delta\langle H_{int}\rangle = \frac{2\hbar^2 a^2}{\pi e^2 d_{int}}\Delta\alpha_{int}, \tag{6b}$$

where $\alpha_{bl/int}$ and $\Delta\alpha_{bl/int}$ are related to $\sigma_{bl/int}(\omega)$ in the same way as α and $\Delta\alpha$ are to $\sigma_c(\omega)$. A schematic representation of $\Delta\alpha_{bl}$ and $\Delta\alpha_{int}$ is shown in Fig. 1. The quantity $\Delta\alpha$ can also be expressed in terms of $\Delta\alpha_{bl}$ and $\Delta\alpha_{int}$. After some manipulations (similar to those in chapter 5.7 of Ref. [13]) using the analytic and the asymptotic properties of $\sigma(\omega)$, $\sigma_{bl}(\omega)$ and $\sigma_{int}(\omega)$ we obtain

$$\alpha(T, \omega \to \infty) = \frac{d_{bl}}{d_{bl} + d_{int}}\alpha_{bl}(T, \omega \to \infty) + \frac{d_{int}}{d_{bl} + d_{int}}\alpha_{int}(T, \omega \to \infty).$$

Provided that the temperature dependence of the three conductivities above Ω_c is negligible we can also write

$$\Delta\alpha = \frac{d_{bl}}{d_{bl} + d_{int}}\Delta\alpha_{bl} + \frac{d_{int}}{d_{bl} + d_{int}}\Delta\alpha_{int}. \tag{7}$$

In order to obtain an estimate of $\Delta\langle H_c\rangle$ we have to make an additional assumption because the Eqs. (4), (6a), (6b) and (7) still contain five unknowns: $\Delta\langle H_c\rangle$, $\Delta\langle H_{bl}\rangle$, $\Delta\langle H_{int}\rangle$, $\Delta\alpha_{bl}$ and $\Delta\alpha_{int}$ ($\Delta\alpha$ is assumed to be known from the infrared data). We suggest to use the following assumption:

$$\frac{\Delta\alpha_{bl}}{\Delta\alpha_{int}} = \frac{\omega_{bl}^2}{\omega_{int}^2}, \tag{8}$$

where ω_{bl} and ω_{int} are the superfluid plasma frequencies of the intra-bilayer region and the inter-bilayer region, respectively. In other words, we suggest that $\Delta\alpha_{bl/int}$ is proportional to the contribution $\rho_{s\,bl/int}$ of the condensate

(see Fig. 1 for a definition of $\rho_{s\,bl/int}$). Using Eqs. (4), (6a), (6b), (7) and (8) we finally arrive at the formula expressing $\Delta\langle H_c\rangle$ in terms of $\Delta\alpha$:

$$\Delta\langle H_c\rangle = k\,\frac{d_{bl}+d_{int}}{d_{bl}d_{int}}\,\frac{\omega_{bl}^2 d_{int}+\omega_{int}^2 d_{bl}}{\omega_{bl}^2 d_{bl}+\omega_{int}^2 d_{int}}\,\Delta\alpha\,, \tag{9}$$

where $k = 2\hbar^2 a^2/(\pi\,e^2)$. There is one special case, where there is no need to use any additional assumption like Eq. (8), the case of negligible inter-bilayer conductivity. Then $\Delta\alpha_{int}$ can be neglected and we obtain

$$\Delta\langle H_c\rangle = k\,\frac{d_{bl}+d_{int}}{d_{bl}^2}\,\Delta\alpha\ \text{ or equivalently}$$

$$\Delta H_c[\text{meV}] = \frac{4.7\cdot 10^{-5}}{d_{bl}[\text{Å}]}\frac{d_{bl}+d_{int}}{d_{bl}}\Delta\alpha[\Omega^{-1}\text{cm}^{-2}]\,. \tag{10}$$

The following points deserve some comments.

(i) Applicability of Eq. (10): Equation (10) can certainly be used for systems like Bi-2212, where the inter-bilayer conductivity is known to be very small. However, it can still be used—with a precision of about 20%—for the less anisotropic Y-123 because even there the ratio $\omega_{int}^2/\omega_{bl}^2$ as obtained from the infrared data is fairly small, especially in the underdoped samples.

(ii) Comparison with the sum-rule for single layer materials: Note the difference between the right hand sides of Eq. (3) and Eq. (10). The right hand side of Eq. (10) is larger by a factor of $(d_{bl}+d_{int})^2/d_{bl}^2$ than the one of Eq. (3) (with $d = d_{bl}+d_{int}$). It means that the sum-rule in Eq. (3) underestimates $\Delta\langle H_c\rangle$ by a factor of $d_{bl}^2/(d_{bl}+d_{int})^2$ (1/20 for Bi-2212). *For a strongly anisotropic bilayer compound even a tiny change of SW may correspond to a significant change of $\langle H_c\rangle$.*

(iii) Interband polarizability: In the derivation of Eq. (10) we have tacitly assumed that the interband polarizabilities of the two regions, intra-bilayer and inter-bilayer, are the same, i.e., that the two conductivities, σ_{bl} and σ_{int}, contain the same term $-i\omega\varepsilon_\infty$. For $\varepsilon_{\infty\,bl}\neq\varepsilon_{\infty\,int}$ we would obtain

$$\Delta\langle H_c\rangle = k\,\frac{\Delta\alpha_{bl}}{d_{bl}} = k\,\frac{(d_{bl}\varepsilon_{\infty\,int}+d_{int}\varepsilon_{\infty\,bl})^2}{d_{bl}^2(d_{bl}+d_{int})\varepsilon_{\infty\,int}^2}\,\Delta\alpha\,. \tag{11}$$

(iv) SR for multilayer cuprates with $n>2$: The approach presented above can easily be generalized and we obtain

$$\Delta\langle H_c\rangle = k\,\frac{(n-1)d_{bl}+d_{int}}{d_{bl}^2}\,\Delta\alpha\,. \tag{12}$$

The distance between the closely-spaced copper-oxygen planes is denoted by d_{bl}, d_{int} is the distance between the multilayer blocks. In order to obtain Eq. (12) we have assumed that the inter-multilayer conductivity is negligible and that all the "Josephson junctions" (regions between neighboring copper-oxygen planes) within the multilayer block exhibit the same electronic conductivity.

In Table I we show the values of ρ_s, ΔN, $\Delta\alpha = \rho_s + \Delta N$, $\Delta\langle H_c\rangle$, E_J, and the condensation energy per unit cell (U_0) for Y123, Bi-2212, and Bi-2223.

$\underline{\rho_s}$ The values of ρ_s for Y-123 have been obtained by using Eq. (A3) of Ref. [5] and the values of ω_{bl} and ω_{int} presented in Refs. [8,9]. The corresponding values of the c-axis plasma frequency are 250, 520, and 1500 cm^{-1}. The values of ρ_s in the Bi-compounds are negligibly small.

$\underline{\Delta N}$ $\Delta N = N_s(\Omega_c) - N_n(\Omega_c)$, $N_s(\omega) = N(T \ll T_c,\omega)$, $N_n = N(T\approx T_c,\omega)$, $N(T,\omega) = \int_{0^+}^\omega \sigma_{1c}(T,\omega')\,d\omega'$. The values of ΔN for underdoped Y-123 have been obtained by integrating the conductivity data presented in Refs. [8,14] supplemented by mid-infrared data ranging up to $\Omega_c = 1500$ cm^{-1} that have been more recently obtained by ellipsometric measurements. A linear extrapolation of σ_{1c} below 100 cm^{-1} has been used. Figure 2(a) shows a complete set of ellipsometric data for the $T_c = 80$ K sample including the extrapolation. Note that $\sigma_{1c}(T = 20\,\text{K} \ll T_c)$ is larger than $\sigma_{1c}(T = 100\,\text{K} \approx T_c)$ in two different regions: in the region around 550 cm^{-1} (label B in Fig. 2(a)) and in the one located above the frequency range of the apical oxygen modes and centered at 900 cm^{-1} (label C). The additional absorption band B has already been attributed [8,9] to the transverse plasma excitation (TPE) [7–9,11]. We propose that the band C also belongs to the TPE since a splitting of the SW of the TPE into two parts—one below and one above the frequency range of the apical-oxygen modes—is consistent with the Josephson superlattice model (JSM) [7–9,11]. This is demonstrated in the inset of Fig. 2(a) which shows results of the model calculations of σ_{1c} for two sets of parameters, one corresponding to the SC state and one to the normal state. Details of the calculations are

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given in Ref. [15]. The frequency dependence of the quantity $N_n(\omega) - N_s(\omega)$ corresponding to the data of Fig. 2(a) is shown in Fig. 2(b). Note that our value of $|\Delta N|$ of $0.5\rho_s$ is considerably smaller than the one of $0.8\rho_s$ obtained by Basov $et\ al.$ [2] for samples with a similar value of T_c. This is mainly due to the fact that the band C appears above the value of Ω_c of Ref. [2] (800 cm^{-1}) while below our value of 1500 cm^{-1}. The value of ΔN for optimum doped Y-123 has been estimated from the inset of Fig. 1 of Ref. [9]. The difference with respect to the result presented in Ref. [2] is again related to the difference in the value of the cutoff frequency Ω_c. The values of ΔN for Bi-2212 and Bi-2223 are taken from Refs. [5] and [16], respectively.

$\underline{\Delta\langle H_c \rangle}$ The values of $\Delta\langle H_c \rangle$ result from Eq. (9) (Y-123), Eq. (10) (Bi-2212), and Eq. (12) (Bi-2223).

$\underline{U_0}$ The values of U_0 have been obtained from the specific heat data in Ref. [17] (Y123) and in Ref. [18] (Bi-2212), the value for Bi-2223 is an estimated upper limit, $U_0(\text{Bi} - 2223) \approx 2U_0(\text{Bi} - 2212)$.

In all the compounds studied $\langle H_c \rangle$ decreases below T_c and in all of them except for Bi-2212 $\Delta\langle H_c \rangle$ is of the same order of magnitude as U_0. The values of $\Delta\langle H_c \rangle$ in Y-123 are smaller (ca by a factor of 2) than those of E_J presented in Ref. [5], whereas according to the simplest version of the JSM, neglecting the single particle tunnelling, they should be larger by a factor of 4 [4,5]. This is related to the fact that the values of the SW change $\Delta\alpha$ are considerably smaller (ca by a factor of 8) [19] than the estimates of the SW of the TPE based on the JSM. We are aware of the following reasons for this discrepancy.

(i) Non zero single particle contribution, i.e., $\Delta N_{int} < 0$ and $\Delta N_{bl} < 0$ (see Fig. 1 for a definition of ΔN_{int} and ΔN_{bl}). Physically this means that parts of $\rho_{s\,int}$ and $\rho_{s\,bl}$ originate from the low-frequency regions of $\sigma_{int\,1}$ and $\sigma_{bl\,1}$, respectively. As a consequence, the SR-based estimate of $\Delta\langle H_c \rangle$ determined by the changes of the total FIR SWs, $\Delta\alpha_{int} < \rho_{s\,int}$ and $\Delta\alpha_{bl} < \rho_{s\,bl}$, (see Eqs. (6a) and (6b)) is smaller than the JCE estimate determined solely by $\rho_{s\,int}$ and $\rho_{s\,bl}$. This is probably the main reason for the discrepancy.

(ii) Finite compressibility (FC) effects. The latter have been shown [11] to shift both the frequency of the (longitudinal) intra-bilayer plasmon and the frequency of the TPE towards higher values with respect to the "bar values" of the simplest version of the JSM. At the same time the FC effects do not influence the SW of the TPE. Since the effects have not been considered when analyzing the data in Refs. [8–10,5], the resulting values of ω_{bl} presented therein, and collected in Ref. [5], may be a bit higher than the actual ones. This would imply that also the values of E_J in Ref. [5] are higher than the actual ones, which would account at least for a part of the discrepancy. We do not think, however, that this is the main reason for it. If the influence of the FC effects was considerable, the frequency of the TPE in Bi-2223 would be significantly lower than in Bi-2212 because the FC induced shift of the plasma frequencies decreases with increasing distance between the outer copper-oxygen planes of the multilayer block. This has not been observed [10,5,16].

(iii) "Pseudogap below T_c". The difference between the results of the two approaches (JCE and SR) could be much smaller, if we considered the temperature evolution of the normal state spectra below T_c, i.e., if $\Delta\alpha$ was taken as $\alpha(T << T_c, \Omega_c) - \lim_{T\to0}\alpha(\text{normal state}, T, \Omega_c)$. It is this limit that should be actually used when estimating $\Delta\langle H_c \rangle$ as emphasized also elsewhere [2].

(iv) Fluctuation effects above T_c. In strongly underdoped Y-123 the additional absorption peak starts to form already at temperatures much higher than T_c [20,14], presumably due to fluctuation effects [8,5]. The JCE-based estimate of $\Delta\langle H_c \rangle$ contains a contribution of these effects since it is determined by the low temperature value of ω_{bl} (i.e., in a way that does not require any assumptions concerning the onset of superconductivity). On the other hand the effects obviously do not contribute to the SR-based estimate determined by the change of the spectra below T_c.

The SW of the additional absorption band corresponding to the TPE ($\Delta\alpha$ in Table 1) is suprisingly small in Bi-2212 as compared to Y-123. We suggest that this is largely due to the fact that the spacing layer in Bi-2212 is more insulating than that of Y-123. Indeed, it can be seen from Eq. (11) that for a given value of $\Delta\alpha_{bl}$ a decrease of the ratio $\varepsilon_{\infty\,int}/\varepsilon_{\infty\,bl}$ leads to a reduction of $\Delta\alpha$. A similar reduction of $\Delta\alpha$ can also be caused by the electronic background of the intra-bilayer region: this is what has been assumed in Refs. [10,5]. These observations indicate that the estimate of $\Delta\langle H_c \rangle$ in Bi-2212 based on Eq. (10) may be considerably lower than the actual value of this quantity. Furthermore, it can not be excluded yet that a part of $\Delta\alpha$ appears at frequencies above the FIR range. At present we do not know why $\Delta\alpha$ is almost by a factor of 5 larger in Bi-2223 than in Bi-2212 (the JSM yields a factor of ca 2).

In summary, we have developed an extension of the c-axis tight-binding SR that applies to multilayer HTCS and allows one to estimate—model independently— the kinetic energy change $\Delta\langle H_c \rangle$ associated with the SC transition. For multilayer HTCS with insulating (or almost insulating) spacing layers the ratio between the SW change and $\Delta\langle H_c \rangle$ is determined by a geometrical factor that is typically an order of magnitude lower than the one of the conventional tight-binding SR. Using published far-infrared data that are in part complemented by new MIR data for underdoped Y-123, we found that $\langle H_c \rangle$ decreases below T_c in all the compounds studied including optimally doped Y-123 and almost optimally doped Bi-2212 and Bi-2223. The decrease seems thus not to be restricted to the underdoped regime. In all the compounds studied except for Bi-2212 $\Delta\langle H_c \rangle$ as determined by the SR is of the same order of magnitude as the condensation energy and there are several reasons to believe that for Bi-2212 $\Delta\langle H_c \rangle$ is underestimated. To

conclude, in all the multilayer HTCS studied the decrease of $\langle H_c \rangle$ below T_c does represent a significant contribution to the condensation energy. Its high value suggests that it may well be the dominant contribution. The possibility, however, that the changes of $\langle H_c \rangle$ below T_c simply parallel much larger changes of the in-plane kinetic energy (such as obtained in Ref. [21] for Bi-2212) cannot be excluded.

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TABLE I. Values of the superfluid spectral weight ρ_s, the quantity ΔN defined in the text, the spectral weight change $\Delta\alpha$, the kinetic energy change $\Delta\langle H_c\rangle$, the Josephson coupling energy E_J as obtained in Ref. [5], and the condensation energy U_0. The quantities ρ_s, ΔN, and $\Delta\alpha$ are given in $m\Omega^{-1}cm^{-2}$, the quantities $\Delta\langle H_c\rangle$, E_J, and U_0 in meV.

compound	$T_c[K]$	ρ_s	ΔN	$-\Delta N/\rho_s$	$\Delta\alpha$	$\Delta\langle H_c\rangle$	E_J	U_0	$\Delta\langle H_c\rangle/U_0$
$YBa_2Cu_3O_{6.55}$	53	1.7	0	0	1.7	0.08	0.13	0.05	1.6
$YBa_2Cu_3O_{6.75}$	80	7.2	-3.5	0.5	3.7	0.16	0.30	0.16	1.0
$YBa_2Cu_3O_{6.93}$	91	60.0	-48.0	0.8	12.0	0.47	1.14	0.36	1.3
$Bi_2Sr_2CaCu_2O_8$	91	0	0.3	—	0.3	0.02	0.13	0.13	0.15
$Bi_2Sr_2Ca_2Cu_3O_{10}$	107	0	1.4	—	1.4	0.11	-	0.26	0.4

FIG. 1. Schematic representation of (a) the bilayer geometry and (b) the spectra of $\sigma_{int\,1} = Re\{\sigma_{int}\}$ and $\sigma_{bl\,1} = Re\{\sigma_{bl}\}$ and the quantities that describe the related spectral weight changes (as discussed in the text).

FIG. 2. (a) Experimental spectra of the c-axis conductivity of slightly underdoped ($T_c = 80\,K$) Y-123. The labels A, B, and C indicate the region of a pronounced gap-like suppression of σ_{1c}, the main part of the additional absorption band due to the transverse plasma excitation, and its high frequency satellite, respectively. Inset: results of model calculations for the superconducting state (solid line) and for the normal state (dashed line) demonstrating the splitting of the additional absorption band. (b) Spectra of the quantity $N_n - N_s$ defined in the text.
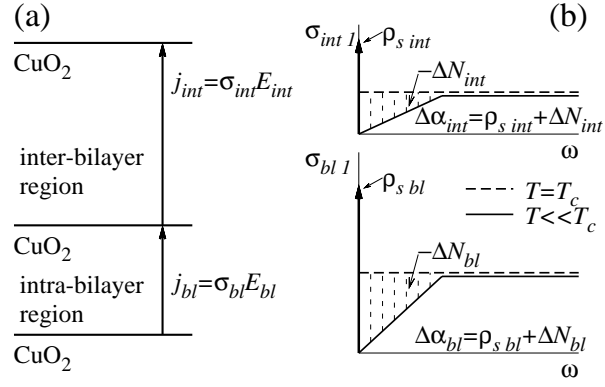
Fig. 1

(a)

$\overline{\text{CuO}_2}$

$j_{int}=\sigma_{int}E_{int}$

inter-bilayer region

$\overline{\text{CuO}_2}$

intra-bilayer region

$j_{bl}=\sigma_{bl}E_{bl}$

$\overline{\text{CuO}_2}$

(b)

$\sigma_{int\,1}$ $\rho_{s\,int}$

$-\Delta N_{int}$

$\Delta\alpha_{int}=\rho_{s\,int}+\Delta N_{int}$

ω

$\sigma_{bl\,1}$ $\rho_{s\,bl}$

$----$ $T=T_c$

$\underline{\quad\quad}$ $T<<T_c$

$-\Delta N_{bl}$

$\Delta\alpha_{bl}=\rho_{s\,bl}+\Delta N_{bl}$

ω

Fig. 2